

DIVISION



STAGE 1

SUMMARY –

Children are taught to understand division as 'repeated subtraction', using sharing and grouping.

VOCABULARY

Share, share equally, one each, two each..., group, groups of, lots of.

IMAGES

Practical experience of sharing

10 fat sausages sizzling in the pan - encourages counting back in 2s.
Putting things into pairs

Putting things into pairs



Although division is not formally introduced at this Stage, the ground work is vital in Foundation Stage.

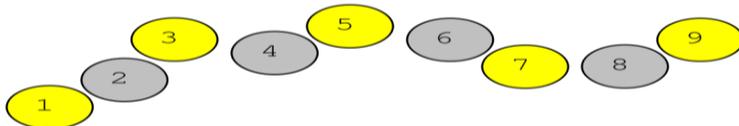
This includes songs that include equal jumps.

They can share toys, fruit and other materials out.

KEY SKILLS

- Solve one step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- Through grouping and sharing small quantities, pupils being to understand, division of numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

Counting on in 'groups'



Can we jump in 2s along a number track? Will we land on 7? Why not?



Giving visual images for division is important.
Sorting objects and people into groups is the main skill taught.

RESOURCES

Children should still have access to wide range of resources such as counting equipment, everyday objects, number tracks, number lines, Numicon, counting beads, socks, children etc.

DIVISION



STAGE 2

SUMMARY –

Children are taught to understand division as 'repeated subtraction', using sharing and grouping.

IMAGES

Practical experience of sharing

10 fat sausages sizzling in the pan - encourages counting back in 2s.
Putting things into pairs

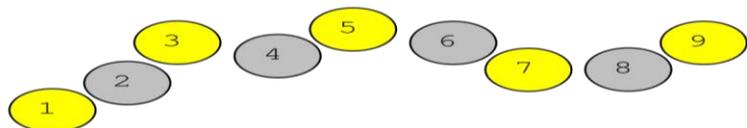
Putting things into pairs



This includes songs that include equal jumps.

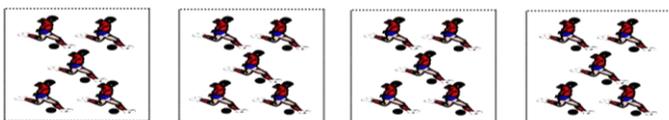
They can share toys, fruit and other material out.

Counting on in 'groups'



Can we jump in 2s along a number track? Will we land on 7? Why not?

Introducing groups



20 children get into teams of 5 to play a game.
How many teams are there?

Division is known as **repeated Subtraction** but is also opposite (inverse) of multiplication.
 $3 \times 3 = 9$
 $9 \div 3 = 3$

Giving visual images for division is Important. Sorting objects and people into groups is the main skill taught.

VOCABULARY

Share, share equally, one each, two each..., group, groups of, lots of.

KEY SKILLS

- Solve one step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- Through grouping and sharing small quantities, pupils being to understand, division, and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

RESOURCES

Children should still have access to wide range of resources such as counting equipment, everyday objects, number tracks, number lines, numicon, counting beads, socks, children etc.

DIVISION

SUMMARY – Children are taught to understand division as 'repeated subtraction', using sharing and grouping.

IMAGES

Introducing sharing
(compared to grouping)

$6 \div 2$ can be solved in two practical ways.

Sharing

$$6 \div 2 = \square$$

6 lollies are shared between 2 children. How many lollies does each child get?



Sharing

Share 6 sweets between 2 children.

Grouping

Repeated subtraction – how many groups of two are there in 6?

Initially sharing is a powerful image for the children to use.

However, when numbers increase this can no longer be carried out practically.

It is important that when grouping is used the children make links with counting in groups on a number line.

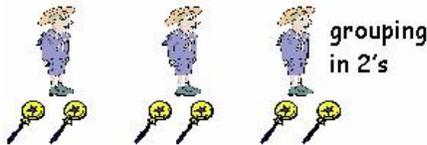
They are also encouraged to count up using multiplication facts. (repeated addition)

As confidence grows the children use facts to find 'missing numbers'.

(In this example there are 5 jumps of 3 in 15. $15 \div 3 = 5$ and $15 \div 5 = 3$)

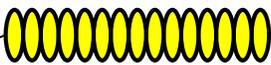
Grouping

There are 6 lollies. How many children can have two each?

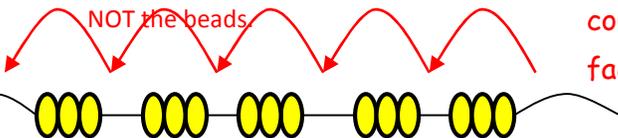


More grouping

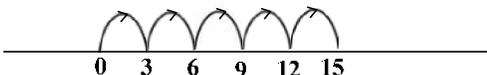
How many 3s in 15?



Remember to count the gaps,



Here the beads are sorted into groups of 3.



How many 3s in 15?



Using a number line, children count on in 3s until 15 is reached. They then count how many jumps of 3 they have made.



STAGE 3A

VOCABULARY

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over remainder.

KEY SKILLS

- Count in steps of 2,3 and 5 from 0
- Recall and use multiplication and division facts for the 2,5 and 10 multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the \times , \div and $=$ signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in context.

RESOURCES

Children should still have access to wide range of resources such as counting equipment, everyday objects, number tracks, number lines, numicon, counting beads etc.

DIVISION



STAGE 3B

VOCABULARY

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry' remainder, multiple, Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over remainder.

KEY SKILLS

- Recall and use multiplication and division facts for the 2,3,4,5 and 10 multiplication (through doubling).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplications and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (eg. Using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts ($30 \times 2 = 60$, so $60 \div 3 = 20$ and $20 = 60 \div 3$).
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

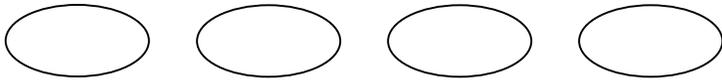
SUMMARY – Children are taught to understand division through practical sharing looking at sharing 2 digit numbers using number lines.

IMAGES

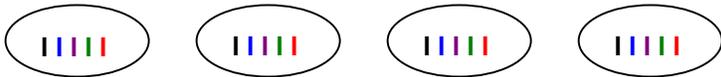
$$20 \div 4 =$$

Step 1: Understand the question means
'What is 24 shared equally into 4 sets?'

Step 2: Draw the correct number of sets.



Step 3: Share out the amount you have, by counting on, starting from 1, putting a mark I each set as you go.



Step 4: count how many in each set.

$$\text{So } 20 \div 4 = 5$$

Children will also be shown the link between finding a half and dividing by 2 or finding a $\frac{1}{4}$ by dividing by 4.

eg. $\frac{1}{2}$ of 6 is 3 or $6 \div 2 = 3$
 $\frac{1}{4}$ of 20 is 5 or $20 \div 4 = 5$

This method builds on a lot of practical sharing experience, manipulating objects/counters. Once mastered, the children can replicate the process on paper, using dots or tally marks within quickly drawn sets.

RESOURCES

Children should still have access to wide range of resources such as counting equipment, everyday objects, number tracks, number lines, numicon, counting beads etc.

DIVISION



STAGE 4A

VOCABULARY

Share, share equally, one each, two each..., group, groups of, array, lots of, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry' remainder, multiple.

KEY SKILLS

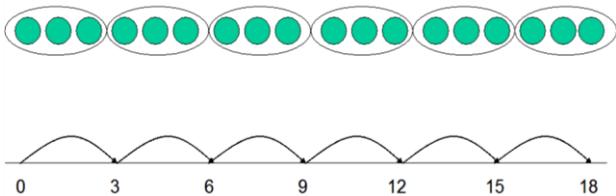
- Recall and use multiplication and division facts for the 2,3,4,5,8 and 10 multiplication (through doubling, connect the 2,4 and 8s)
- Halve numbers up to 100.
- Understand that a remainder is the amount left over after a division.
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers.
- Solve problems, in contexts, and including missing number problems, involving multiplications and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (eg. Using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts ($30 \times 2 = 60$, so $60 \div 3 = 20$ and $20 = 60 \div 3$).
- Progress to using formal written methods, chunking on a number line, to divide numbers beyond the range of known number facts.

SUMMARY – Children are taught to understand division through practical sharing looking at sharing 2 digit numbers using number lines.

IMAGES

Using number lines

$$18 \div 3 =$$



It is important that children can make a link between grouping and jumping on a number line.

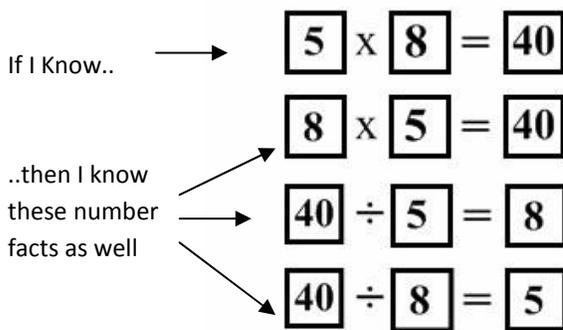
Children need to understand the link between division and multiplication (they are exact opposites - the inverse of each other)

They will be taught division facts alongside multiplication.

At this stage children are beginning to recite multiplication and division facts (2, 3, 4, 5, 8, 10).

Children may use addition or subtraction when grouping. This example shows how counting on in 3s helps solve $18 \div 3 = 6$. This is supported with some hands on grouping of objects.

'3 facts for free'



Using multiplication to solve division problems

$$20 \div _ = 2$$

$$_ \div 10 = 3$$

RESOURCES

Children should still have access to wide range of resources such as counting equipment, everyday objects, number tracks, number lines, numicon, counting beads etc.

DIVISION



STAGE 4B

VOCABULARY

Share, share equally, one each, two each..., group, groups of, array, lots of, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry' remainder, multiple.

KEY SKILLS

- Recall and use multiplication and division facts for the 2,3,4,5,8 and 10 multiplication (through doubling, connect the 2,4 and 8s)
- Double and halve numbers up to 100.
- Understand that a remainder is the amount left over after a division.
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers.
- Solve problems, in contexts, and including missing number problems, involving multiplications and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (eg. Using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts ($30 \times 2 = 60$, so $60 \div 3 = 20$ and $20 = 60 \div 3$).
- Progress to using formal written methods, chunking on a number line, to divide numbers beyond the range of known number facts.

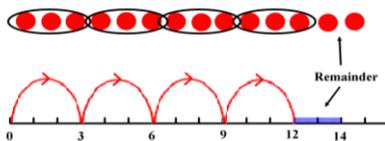
SUMMARY –

Children are taught to understand division through number lines including remainders.

IMAGES

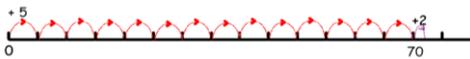
Using number lines for larger numbers or to show remainders.

$$14 \div 3 = 4 \text{ remainder } 2$$



The children by now are using number lines, counting on in small 'jumps'. In this example 14 jumps of 5 have been completed, leaving a remainder of 2.

$$72 \div 5$$



Using the language of 'remainder';

Grouping objects and drawing number lines gives a good visual understanding of remainders.

Children will need to be familiar with interpreting the remainders, when faces with real life problems

e.g. A box of cans holds 4 cola bottles

How many boxes do I need if I want 18 bottles?

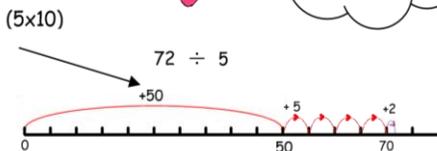
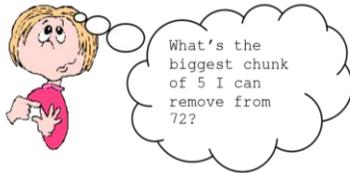
This method relies on good understanding of multiplication tables to be able to recognise simple multiplication facts that will help solve a division problem.

e.g. to know that $5 \times 10 = 50$ as a way Of breaking down $72 \div 5 = _$

Here one large jump to 50 has been Made because the child understands That jumping 50 is the same as jumping in 5's 10 times.

Using the number line confidently

The next step to speed things up is to make larger 'jumps'...



Children will also be shown how to count back on a number line to solve the same problem.

RESOURCES

Children should still have access to wide range of resources such as counting equipment, everyday objects, number tracks, number lines, numicon, counting beads etc.

DIVISION

- Recall multiplications and division facts for all numbers up to 12×12
- Multiply and divide numbers mentally, drawing upon known facts
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers. Solve problems involving multiplication and division where larger numbers are decomposed into their factors
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- Work out whether a number up to 100 is prime, and recall numbers to 10
- Divide numbers up to 4 digits by a one-digit number using the normal written method of short division and interpret remainders appropriately or the context
- Use multiplication and division as inverses
- Interpret no-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (eg. $98 \div 4 = 24 \text{ r } 2 = 24 \frac{1}{2} = 24.5 \approx 25$)
- Solve problems involving combinations of all four operations, including understating of the equals sign

SUMMARY –

Children are taught to 'chunk'. Chunking is a method used for dividing larger numbers that cannot be divided mentally. Chunking is repeated subtraction of the divisor and multiples of the divisor - in other words, working out how many groups of a number fit into another number. Children are then taught chunking with remainders.

IMAGES

155
 $- 50$ (10 × 5)
 105
 $- 50$ (10 × 5)
 55
 $- 50$ (10 × 5)
 5
 $- 5$ (1 × 5)
 0

31 groups of 5 have been subtracted

Therefore $155 \div 5 = 31$

$73 \div 5$ How many 5s make 73?

$$\begin{array}{r} 73 \\ - 50 \quad (10 \times 5) \\ \hline 23 \\ - 20 \quad (4 \times 5) \\ \hline 3 \end{array}$$

How many 5s have been subtracted?
 14 sets of 5, with 3 left over.

$$73 \div 5 = 14 \text{ r}3$$

RESOURCES

Children should still have access to wide range of resources such as counting equipment, everyday objects, number tracks, number lines, numicon, counting beads etc.

STAGE 5

VOCABULARY

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over remainder, inverse, short division, 'carry' remainder, multiple, divisible by, factor, chunk

KEY SKILLS

- Recall multiplications and division facts for all numbers up to 12×12
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers. Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- Work out whether a number up to 100 is prime, and recall numbers to 19
- Divide numbers up to 4 digits by a one-digit number using the normal written method of short division and interpret remainders appropriately or the context
- Use multiplication and division as inverses
- Interpret no-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (eg. $98 \div 4 = 24 \text{ r } 2 = 24 \frac{1}{2} = 24.5 \approx 25$)
- Solve problems involving combinations of all four operations, including understating of the equals sign.

DIVISION



STAGE 6

VOCABULARY

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over remainder, inverse, short division, 'carry' remainder, multiple, divisible by, factor & common factor

KEY SKILLS

- Recall and use multiplication and division facts for all numbers to 12×12 for more complex calculations
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method as long division, and interpret remainders as whole numbers remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers
- Solve problems involving all 4 operations
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem
- Use written division methods in cases where the answers has up to two decimal places
- Solve problems which require answers to be rounded to specific degrees of accuracy.

SUMMARY –

Children should begin to be able to use a more compact written method. They at this stage should be taught

'The Bus Stop or short division method.' Including decimals

IMAGES

$$\begin{array}{r} 120 \\ 8 \overline{) 960} \\ \underline{80} \\ 160 \\ \underline{160} \\ 0 \end{array}$$

If a decimal answer is wanted, step 4 could look like this.

$$\begin{array}{r} 044. \\ 8 \overline{) 356.40} \\ \underline{24} \\ 116 \\ \underline{112} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

↓

$$\begin{array}{r} 044.5 \\ 8 \overline{) 356.40} \\ \underline{24} \\ 116 \\ \underline{112} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Note 365 has now been written as 365.0, the remainder of 4 is put in front of the 0 and then 40 is divided by 8.

RESOURCES

Children should still have access to wide range of resources such as counting equipment, everyday objects, number tracks, number lines, numicon, counting beads Bus Stop Method, Abacus etc.